Econometrics

Homework 4

Problem 1. Suppose that $X_1, ..., X_n$ is an i.i.d. random sample from the uniform distribution on $[0, \theta]$, where $\theta > 0$ is unknown. Denote $X_{(n)} = \max\{X_1, ..., X_n\}$. Let $Q = X_{(n)}/\theta$ and $c_n = \alpha^{1/n}$. Show that $\Pr[Q \le c_n] = \alpha$. Note that $\Pr[Q \le 1] = 1$ and therefore, $\Pr[c_n \le Q \le 1] = 1 - \alpha$. Use this result to show that $\left[X_{(n)}, X_{(n)}/c_n\right]$ is a valid $1 - \alpha$ confidence interval for θ:

$$\Pr\left[\theta \in \left[X_{(n)}, X_{(n)}/c_n\right]\right] = 1 - \alpha.$$

Problem 2. Find A-C in the Stata output below if the number of observations is 23. Justify your answer. According to the results, is x significant? The significance level is 5%. Explain.

y	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
x	.0910863	A	1.32	0.191	B	C
_cons	.3599422	.0241652	14.90	0.000	.3096879	.4101965

Problem 3. The variable *rdintens* is expenditures on research and development (R&D) as a percentage of sales. Sales are measured in millions of dollars. The variable *profmarg* is profits as a percentage of sales.

Using the data for 32 firms in the chemical industry, the following equation is estimated:

$$\begin{aligned} \widehat{rdintens} &= 0.472 + 0.321 \log(sales) + 0.050 profmarg \\ & \quad (1.369) \quad (0.216) \qquad\qquad (0.046) \\ & \quad n = 32,\ R^2 = 0.099. \end{aligned}$$

1. Interpret the coefficient on log($sales$). In particular, if $sales$ increases by 10%, what is the estimated percentage point change in $rdintens$? Is this an economically large effect?

2. Test the hypothesis that R&D intensity does not change with $sales$ against the alternative that it does increase with sales. Do the test at the 5% and 10% levels.

3. Interpret the coefficient on $profmarg$. Is it economically large?

4. Does $profmarg$ have a statistically significant effect on $rdintens$?

Problem 4. Suppose we observe a random sample $\{(Y_i, D_i)\}_{i=1}^n$, where Y_i is the dependent variable and D_i is a binary independent variable: for all $i = 1, 2, ..., n$, $D_i = 1$ or $D_i = 0$. Suppose we regress Y_i on D_i with an intercept. Show: the LS estimate of the slope is equal to the difference between the sample averages of the dependent variable of the two groups, observations with $D_i = 1$ and observations with $D_i = 0$. Hint: The sample average of Y of observations with $D_i = 1$ can be written as $\frac{\sum_{i=1}^n D_i Y_i}{\sum_{i=1}^n D_i}$. What is the sample average of Y of observations with $D_i = 0$? Also note: $D_i = D_i^2$.

Problem 5. We are interested in explaining a worker's wage in terms of the number of years of education (educ) and years of experience (exper) using the following model:

$$log(wage) = \beta_0 + \beta_1 educ + \beta_2 exper + u.$$

The estimated parameters by OLS for a sample of $n = 935$ observations are displayed in the table. Several extensions of this model were considered to address the effects of being married (with the binary variable married) and/or being black (with binary black) or possible nonlinearity on the effect of years of experience.

(a) Test whether the wage regressions for married workers and unmarried workers are the same. Hint: Perform a F test. Model 2 is the unrestricted and Model 1 is the restricted.

(b) Based on a statistical test, do the effects of education and experience depend on the marriage status? Hint: Perform a F test. Model 2 is the unrestricted and Model 3 is the restricted.

(c) What we conclude about the possible nonlinearity of the relationship of $log(wage)$ with respect to the years of experience? Can you conclude that years of experience has no significant effect on $log(wage)$ in Model 5? Make two statistical tests to answer these questions. Hint: Look at Model 5 and Model 6. Use a t test to answer the first question and use a F test to answer the second question.

Critical Values		
$F_{2,N,1-\alpha} = 3.00$	$F_{3,N,1-\alpha} = 2.60$	$F_{6,N,1-\alpha} = 2.10$
$t_{12,1-\alpha/2} = 2.18$	$t_{27,1-\alpha/2} = 2.05$	$t_{21,1-\alpha/2} = 2.08$
$\alpha = 0.05$, $N > 200$		

Variables	Model 1	Model 2	Model 3	Model 5	Model 6
educ	0.07778	0.05316	0.07815	0.071984	0.05571
	(0.00669)	(0.02085)	(0.00653)	(0.00677)	(0.00600)
exper	0.01977	0.00038	0.01829	0.01678	
	(0.00330)	(0.01066)	(0.00330)	(0.01389)	
educ×married		0.02813			
		(0.02194)			
exper×married		0.01952			
		(0.01120)			
married		-0.38069	0.20926	0.18873	0.21311
		(0.36818)	(0.04272)	(0.04763)	(0.04709)
black				-0.24128	-0.22500
				(0.08417)	(0.08212)
married×black				0.03543	0.01071
				(0.09404)	(0.09224)
$exper^2$				0.0000486	
				(0.00058)	
constant	5.50271	5.85694	5.32796	5.46653	5.86609
	(0.11427)	(0.34889)	(0.11574)	(0.12914)	(0.09445)
observations	935	935	935	935	935
R^2	0.13086	0.15705	0.15420	0.18132	0.15417